

2026 Summer Math Packet for Incoming Math 8 Students

Welcome to the 2026-2027 school year! The attached packet has been developed to help you prepare for eighth grade math. Please space out your work on the packet, making a goal of **20 minutes per day, three days per week** on average. The packets will be collected the first week of school, and will count as an assessment grade for fall trimester. Packets will be graded for effort, not full completion or perfect accuracy.

Please spend the time needed to do a quality job on this packet. Show and organize your work for each problem. Use a calculator where indicated, but write down your calculations and show all of your work!

If you don't remember how to solve a particular type of math problem, please use Khan Academy and IXL as resources. Both have excellent tutorials.

Enjoy your summer vacation, and keep your education moving forward during this break.

For the start of eighth grade math, you will need a 3 subject notebook for notes, a folder for handouts, pencils and a **large eraser** for daily work, and a calculator (preferably TI-30Xa, but another scientific calculator will be fine).

If you have any questions, I am available at hgeiser@stlukesri.org.

Best wishes,
Mrs. Geiser

Unit: Knowledge of Algebra, Patterns, and Functions

Objective: Write equations and Inequalities - B

An **inequality** is a mathematical sentence that contains the symbols $<$, $>$, \leq , or \geq .

Words	Symbols
m is greater than 7.	$m > 7$
r is less than -4 .	$r < -4$
t is greater than or equal to 6.	$t \geq 6$
y is less than or equal to 1.	$y \leq 1$

Examples:

- 1) Two times a number is greater than 10 $2x > 10$
- 2) Three less than a number is less than or equal to 7. $x - 3 \leq 7$
- 3) The sum of a number and 1 is at least 5. $x + 1 \geq 5$
- 4) Cody has \$50 to spend. How many shirts can he buy at \$16.50 each? $16.50x \leq 50$

Write an inequality for each of the following:

1.) Five times a number is greater than 25.	2.) The sum of a number and 6 is at least 15.
3.) 24 divided by some number is less than 7.	4.) Five dollars less than two times Chris' pay is at most \$124.
5.) In Ohio, you can get your license when you turn 16. Write an inequality to show the age of all drivers in Ohio.	6.) Suppose a DVD costs \$19 and a CD costs \$14. Write an inequality to find how many CDs you can buy along with one DVD if you have \$65 to spend.

On a scale of 1 – 5 (1: Weak, 5: Strong) rate yourself on this section of math: 1 2 3 4 5

Unit: Knowledge of Algebra, Patterns, and Functions

Objective: Determine the unknown in a linear equation with 1 or 2 operations

Remember, equations must always remain balanced.

- If you add or subtract the same number from each side of an equation, the two sides remain equal.
- If you multiply or divide the same number from each side of an equation, the two sides remain equal.

Example 1: Solve $x + 5 = 11$

$$\begin{array}{r} x + 5 = 11 \quad \text{Write the equation} \\ - 5 = - 5 \quad \text{Subtract 5 from both sides} \\ \hline x = 6 \quad \text{Simplify} \end{array}$$



$$\begin{array}{r} x + 5 = 11 \quad \text{Write the equation} \\ 6 + 5 = 11 \quad \text{Replace } x \text{ with } 6 \\ 11 = 11 \checkmark \quad \text{The sentence is true} \end{array}$$

Example 2: Solve $-21 = -3y$

$$\begin{array}{r} -21 = -3y \quad \text{Write the equation} \\ -3 = -3 \quad \text{Divide each side by } -3 \\ \hline 7 = y \quad \text{Simplify} \end{array}$$



$$\begin{array}{r} -21 = -3y \quad \text{Write the equation} \\ -21 = -3(7) \quad \text{Replace the } y \text{ with } 7 \\ -21 = -21 \checkmark \quad \text{Multiply - is the sentence true?} \end{array}$$

Example 3: Solve $3x + 2 = 23$

$$\begin{array}{r} 3x + 2 = 23 \quad \text{Write the equation} \\ - 2 = - 2 \quad \text{Subtract 2 from each side} \\ \hline 3x = 21 \quad \text{Simplify} \\ \frac{3x}{3} = \frac{21}{3} \quad \text{Divide each side by } 3 \\ x = 7 \quad \text{Simplify} \end{array}$$



$$\begin{array}{r} 3x + 2 = 23 \quad \text{Write the equation} \\ 3(7) + 2 = 23? \quad \text{Replace } x \text{ with } 7 \\ 21 + 2 = 23? \quad \text{Multiply} \\ 23 = 23? \quad \text{Add - is the sentence true?} \end{array}$$

1.) Solve $x - 9 = -12$	2.) Solve $48 = -6r$
3.) Solve $2t + 7 = -1$	4.) Solve $4t + 3.5 = 12.5$
5.) It costs \$12 to attend a golf clinic with a local pro. Buckets of balls for practice during the clinic cost \$3 each. How many buckets can you buy at the clinic if you have \$30 to spend?	6.) An online retailer charges \$6.99 plus \$0.55 per pound to ship electronics purchases. How many pounds is a DVD player for which the shipping charge is \$11.94?

On a scale of 1 – 5 (1: Weak, 5: Strong) rate yourself on this section of math: 1 2 3 4 5

Unit: Knowledge of Algebra, Patterns, and Functions

Objective: Solve for the unknown in an inequality with one variable.

An **inequality** is a mathematical sentence that contains the symbols $<$, $>$, \leq , or \geq .

Words	Symbols
m is greater than 7.	$m > 7$
r is less than -4 .	$r < -4$
t is greater than or equal to 6.	$t \geq 6$
y is less than or equal to 1.	$y \leq 1$

Example 1: Solve $v + 3 < 5$

$v + 3 < 5$ Write the inequality

$$\begin{array}{r} -3 \quad -3 \quad \text{Subtract 3 from each side} \\ v + 3 < 5 \\ \hline v < 2 \quad \text{Simplify} \end{array}$$

Check: Try 1, a number less than 2

$v + 3 < 5$ Write the inequality

$1 + 3 < 5$ Replace v with 1

$4 < 5$? Is this sentence true? **yes**

Example 2: Solve $2x + 8 < 24$

$2x + 8 < 24$ Write the inequality

$-8 \quad -8$ Subtract 8 from each side

$\frac{2x}{2} < \frac{16}{2}$ Simplify

$x < 8$ Divide each side by 2

$x < 8$ Simplify

Check: Try 7, a number less than 8

$2x + 8 < 24$ Write the inequality

$2(7) + 8 < 24$ Replace x with 7

$14 + 8 < 24$ Multiply 7 by 2

$22 < 24$? Is the sentence true? **yes**

1. Solve $5y + 1 < 36$

2. Solve $4x - 6 > -10$

3. You have \$80. Jeans cost \$29 and shirts cost \$12. Mom told you to buy one pair of jeans and use the rest of the money to buy shirts. Use this information to write and solve an inequality. How many shirts can you buy?

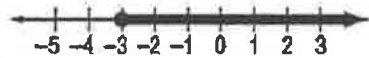
Unit: Knowledge of Algebra, Patterns, and Functions

Objective: Identify or graph solutions of inequalities on a number line.

Examples: Graph each inequality on a number line.



1.) Write an inequality for the graph.

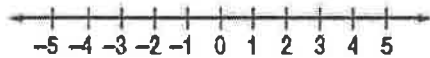


2.) Write an inequality for the graph.



3.) Graph the inequality.

$$b \geq -1$$



4.) Graph the inequality.

$$z < 3$$



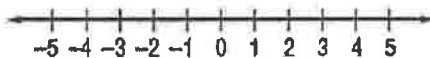
5.) Solve the inequality, then graph it on the number line.

$$y + 9 \leq 13$$



6.) Solve the inequality, then graph it on the number line.

$$4x - 6 > -10$$



On a scale of 1 – 5 (1: Weak, 5: Strong) rate yourself on this section of math: 1 2 3 4 5

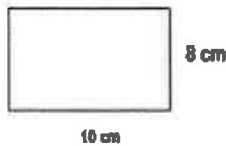
Unit: Knowledge of Algebra, Patterns, and Functions

Objective: Apply given formulas to a problem-solving situation using formulas having no more than three variables.

Example 1:

The perimeter of a rectangle is twice the length (L) plus twice the width (W). $P = 2L + 2W$

Use the given formula to find the perimeter of the rectangle.



$$P = 2L + 2W$$

$$P = 2(10) + 2(8)$$

$$P = 20 + 16$$

$$P = 36 \text{ cm}$$

Write the equation

Replace L and W with the length and width

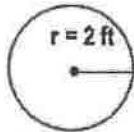
Multiply

Simplify and add the correct label

Example 2:

The area A of a circle equals the product of pi (π) and the square of its radius (r). $A = \pi r^2$ ($\pi \approx 3.14$)

Use the given formula to find the area of the circle.



$$A = \pi r^2$$

$$A = 3.14 \cdot (2)^2$$

$$A = 3.14 \cdot 4$$

$$A = 12.56 \text{ ft}^2$$

Write the equation

Replace π with 3.14 and r with 2

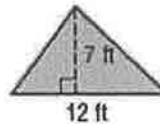
Square the 2

Simplify and add the correct label

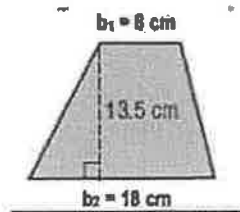
1. The formula for finding the area of a rectangle is $A = L \cdot W$. Use this formula to find the area of the rectangle.



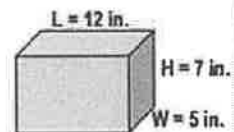
2. The formula for finding the area of a triangle is $A = \frac{1}{2}bh$. Find the area of the triangle below.



3. A trapezoid has two bases (b_1 & b_2). The formula for finding the area of a trapezoid is: $A = \frac{1}{2}h(b_1 + b_2)$. Find the area of the trapezoid.



4. The formula for finding the volume of a rectangular prism is $V = L \cdot W \cdot H$. Find the volume of the box.



5. Margot planted a rectangular garden that was 18 feet long and 10 feet wide. How many feet of fencing will she need to go all the way around the garden? $P = 2L + 2W$

6. Juan ran all the way around a circular track one time. The diameter(d) of the track is 60 meters. The formula for circumference of a circle is $C = \pi d$. Use this formula to find out how far Juan ran.

Unit: Knowledge of Algebra, Patterns, and Functions

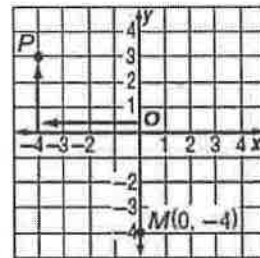
Objective: Graph ordered pairs in a coordinate plane.

The coordinate plane is used to locate points. The horizontal number line is the **x-axis**. The vertical number line is the **y-axis**. Their intersection is the **origin**. Points are located using **ordered pairs**. The first number in an ordered pair is the **x-coordinate**; the second number is the **y-coordinate**. The coordinate plane is separated into four sections called **quadrants**.

Example 1: Name the ordered pair for point P. Then identify the quadrant in which P lies. Quadrant 2 Quadrant 1

- Start at the origin.
- Move 4 units left along the x-axis.
- Move 3 units up on the y-axis.

The ordered pair for point P is $(-4, 3)$.
P is in the upper left quadrant or quadrant II.



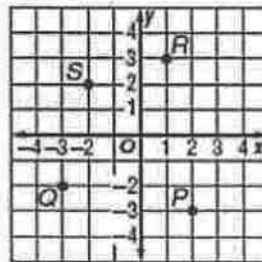
Example 2: Graph and label the point M $(0, -4)$.

- Start at the origin.
- Move 0 units along the x-axis.
- Move 4 units down on the y-axis.
- Draw a dot and label it $M(0, -4)$.

Quadrant 3 Quadrant 4

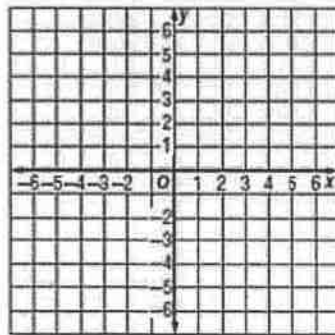
1.) Name the ordered pair for each point graphed at the right. Then identify the quadrant in which each point lies.

Coordinates	Quadrant
P (,)	_____
Q (,)	_____
R (,)	_____
S (,)	_____



3.) Graph and label each point on the coordinate plane.

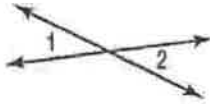
- N $(3, -1)$
- P $(-2, 4)$
- Q $(-3, -4)$
- R $(0, 0)$
- S $(-5, 0)$



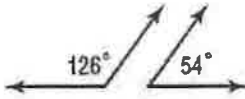
Unit: Knowledge of Geometry

Objective: Identify and describe angles formed by intersecting lines, rays, or line segments - B

Examples:



When two lines intersect, they form two pairs of opposite angles called **vertical angles**, which are always congruent. **Congruent angles** have the same measure. $\angle 1 \cong \angle 2$ means that angle 1 is congruent to angle 2.

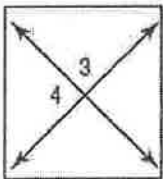


Two angles are **supplementary** if the sum of their measures is 180° . $126^\circ + 54^\circ = 180^\circ$

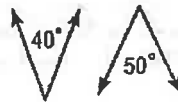


Two angles are **complementary** if the sum of their measures is 90° . $32^\circ + 58^\circ = 90^\circ$

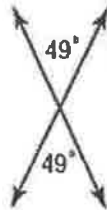
1.) Classify the angles as **complementary**, **supplementary**, or **neither**.



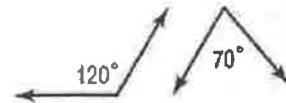
2.) Classify the angles as **complementary**, **supplementary**, or **neither**.



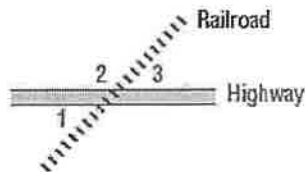
3.) Classify the angles as **complementary**, **supplementary**, or **neither**.



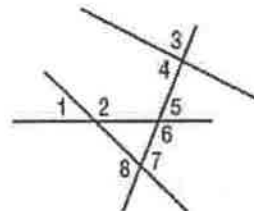
4.) Classify the angles as **complementary**, **supplementary**, or **neither**.



5.) A map shows a railroad crossing a highway, as shown below. Which of the numbered angles are vertical angles?



6.) In a game of pick-up sticks, the last 4 sticks are shown below. Which of the numbered angles are vertical angles?



On a scale of 1 – 5 (1: Weak, 5: Strong) rate yourself on this section of math: 1 2 3 4 5

Unit: Knowledge of Geometry

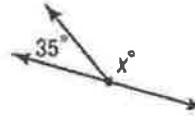
Objective: Determine the measure of angles formed by intersecting lines, line segments, and rays.

Example 1: Find the value of x in the figure.

The two angles are supplementary, so the sum of their measures is 180° .

$$\begin{array}{r} x + 35 = 180 \\ -35 \quad -35 \\ \hline x = 145 \end{array}$$

Write the equation
Subtract 35 from both sides
Simplify
The angle is 145°

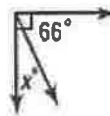


Example 2: Find the value of x in the figure.

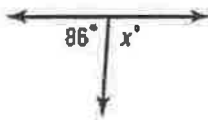
The two angles are complementary, so the sum of their measures is 90° .

$$\begin{array}{r} x + 66 = 90 \\ -66 \quad -66 \\ \hline x = 24 \end{array}$$

Write the equation
Subtract 66 from both sides
Simplify
The angle is 24°



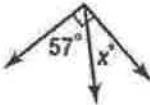
1.) Find the value of x .



2.) Find the value of x .



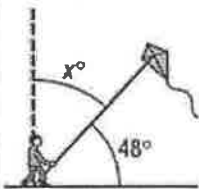
3.) Find the value of x .



4.) Find the value of x .



5.) A kite string makes an angle of 48° with respect to the ground as shown below. The dashed line is vertical and the ground is horizontal. How are the 48° angle and the unknown angle related? What is the value of x ?



6.) A side view of the Great Pyramid at Giza is shown below. The sides of the pyramid make an angle of 52° with respect to the ground. What is the value of x ?

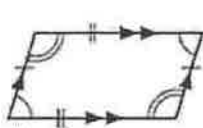


On a scale of 1 – 5 (1: Weak, 5: Strong) rate yourself on this section of math: 1 2 3 4 5

Unit: Knowledge of Geometry

Objective: Determine a missing angle using the sum of the interior angles in a quadrilateral

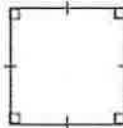
Examples of Quadrilaterals:



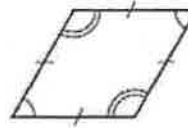
Parallelogram



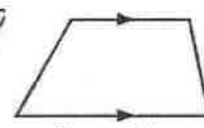
Rectangle



Square



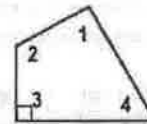
Rhombus



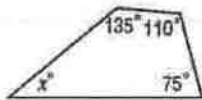
Trapezoid

The sum of the measures of the angles of a quadrilateral is 360° .

$$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360^\circ$$



Example: Find the missing measure in the quadrilateral.



$$135 + 110 + 75 + x = 360$$

$$320 + x = 360$$

$$\begin{array}{r} 320 + x = 360 \\ - 320 \quad - 320 \\ \hline x = 40 \end{array}$$

$$x = 40$$

The sum of the measures is 360°

Simplify

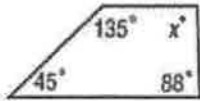
Subtract 320 from each side

The missing angle is 40°

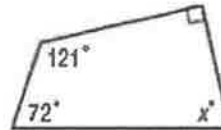
Find the missing measure in each of the following quadrilaterals.

Find the missing measure in each of the following quadrilaterals.

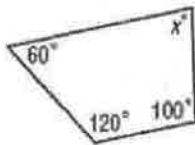
1.



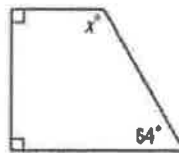
2.



3. The top of Mrs. Blum's coffee table is shown below. Find the measure of the missing angle.



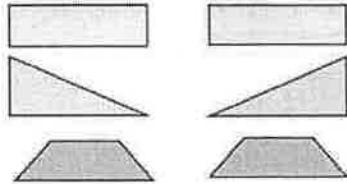
4. Maria needs to cut a piece of carpet to fit the space drawn below. What should the measure of the missing angle be?



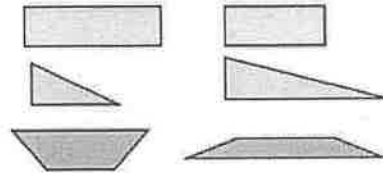
Unit: Knowledge of Geometry

Objective: Determine the congruent parts of polygons.

Congruent Polygons



Non Congruent Polygons



Congruent Polygons	Polygons that have exactly the same size and the same shape
Congruent Segments	Segments that have the same length
Congruent Angles	Angles that have the same measure
Corresponding Sides of a Polygon	Sides of a polygon that are matched up with sides of another congruent or similar polygon
Corresponding Angles of a Polygon	Angles of a polygon that match up with angles of another congruent or similar polygon
$\triangle ABC \cong \triangle DEF$ 	Corresponding sides and angles of congruent polygons are congruent: $\overline{AB} \cong \overline{DE}$ $\angle A \cong \angle D$ $\overline{BC} \cong \overline{EF}$ $\angle B \cong \angle E$ $\overline{AC} \cong \overline{DF}$ $\angle C \cong \angle F$

1.)

Polygon $FGHI \cong$ polygon $NMLK$
 Complete the following congruence statements.
 a) $\overline{GH} \cong$ _____ b) $\overline{KL} \cong$ _____ c) $\overline{FJ} \cong$ _____

2.) Use the figures in problem #1 to complete the following congruence statements.

a) $\angle G \cong$ _____ b) $\angle K \cong$ _____
 c) $\angle H \cong$ _____ d) $\angle F \cong$ _____

3.) Look at the figures in problem #1. Determine the measure of each segment or angle.

a) $x =$ _____ b) $y =$ _____ c) $z =$ _____

4.) Polygon $HJKLMNPQ$ is congruent to polygon $RSTUVXYZ$. What is the length, in units, of \overline{RZ} ? (Note: Figures are not drawn to scale.)

On a scale of 1 – 5 (1: Weak, 5: Strong) rate yourself on this section of math: 1 2 3 4 5

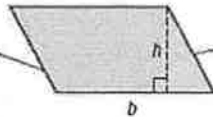
Unit: Knowledge of Measurement

Objective: Estimate and determine the area of quadrilaterals using parallelograms or trapezoids – A.

The area A of a parallelogram equals the product of its base b and its height h . Because rectangles, rhombuses, and squares are all parallelograms, the formula for finding the area of a parallelogram is also used to find the areas of each of these figures.

$A = bh$

The base is any side of a parallelogram.

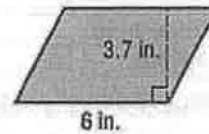


The height is the length of the segment perpendicular to the base with endpoints on opposite sides.

Example: Find the area of a parallelogram if the base is 6 inches and the height is 3.7 inches.

Estimate: $A = 6 \cdot 4$ or 24 in^2

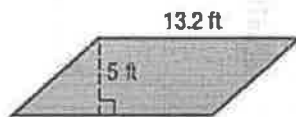
Calculate: $A = bh$ Area of a parallelogram
 $A = 6 \cdot 3.7$ Replace b with 6 and h with 3.7
 $A = 22.2$ Multiply



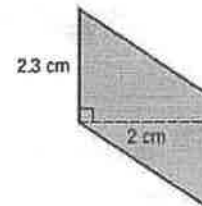
Check: The area of the parallelogram is 22.2 square inches. This is close to the estimate.

Find the area of each parallelogram. Round to the nearest tenth if necessary.

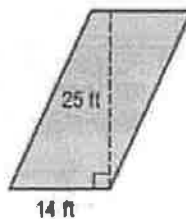
1.)



2.)



3.) Joyce wants to construct a sail with the dimensions shown. How much material will be used?



4.) Two parallel streets are cut across by two other parallel streets as shown in the figure. What is the area of the grassy area in the middle?

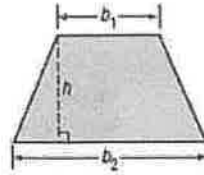


Unit: Knowledge of Measurement

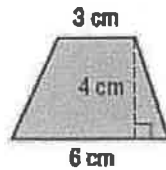
Objective: Estimate and determine the area of quadrilaterals using parallelograms or trapezoids – B.

A trapezoid has two bases, b_1 and b_2 . The height of a trapezoid is the distance between the two bases. The area A of a trapezoid equals half the product of the height h and the sum of the bases b_1 and b_2 .

$$A = \frac{1}{2} h(b_1 + b_2)$$



Example: Find the area of the trapezoid.



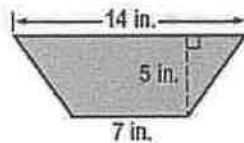
$$A = \frac{1}{2} h(b_1 + b_2)$$
$$A = \frac{1}{2} (4)(3 + 6)$$
$$A = 18$$

Area of a trapezoid
Replace h with 4, b_1 with 3, and b_2 with 6.

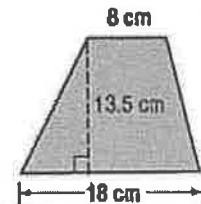
The area of the trapezoid is 18 square centimeters.

Find the area of each trapezoid. Round to the nearest tenth if necessary.

1.)



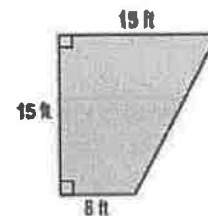
2.)



3. Arkansas has a shape that is similar to a trapezoid with bases of about 182 miles and 267 miles and a height of about 254 miles. Estimate the area of the state.



4. Greta is making a patio with the dimensions given in the figure. What is the area of the patio?



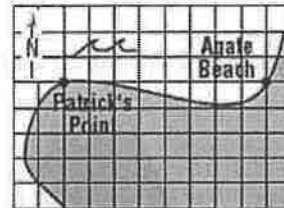
Unit: Knowledge of Measurement

Objective: Determine the distance between 2 points using a drawing and a scale.

A scale drawing represents something that is too large or too small to be drawn at actual size. Similarly, a scale model can be used to represent something that is too large or too small for an actual-size model. The scale gives the relationship between the drawing/model measure and the actual measure.

Example: On this map, each grid unit represents 50 yards. Find the distance from Patrick's Point to Agate Beach.

	Scale		Patrick's Point to Agate Beach	
map	→ 1 unit	=	8 units	← map
actual	→ 50 yards		x yards	← actual



$$1 \cdot x = 50 \cdot 8 \quad \text{cross multiply}$$

$$x = 400 \quad \text{simplify}$$

It is 400 yards from Patrick's Point to Agate Beach.

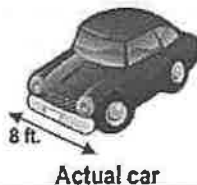
1.) On a map, the distance from Los Angeles to San Diego is 6.35 cm. The scale is 1 cm = 20 miles. What is the actual distance?



2.) Lexie is making a model of the Empire State Building. The scale of the model is 1 inch = 9 feet. The needle at the top is 31.5 feet tall. How big should the needle be on the model?



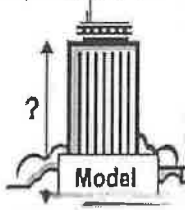
3.) A scale drawing of an automobile has a scale of 1 inch = 1/2 foot. The actual width of the car is 8 feet. What is the width on the scale drawing?



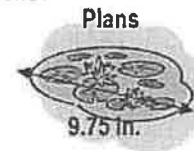
4.) A model ship is built to a scale of 1 cm : 5 meters. The length of the model is 30 centimeters. What is the length of the actual ship?



5.) Jose wants to build a model of a 180-meter tall building. He will be using a scale of 1.5 centimeters = 3.5 meters. How tall will the model be? Round your answer to the nearest tenth.



6.) A pond is being dug according to plans that have a scale of 1 inch = 6.5 feet. The maximum distance across the pond is 9.75 inches on the plans. What will be the actual maximum distance across the pond?



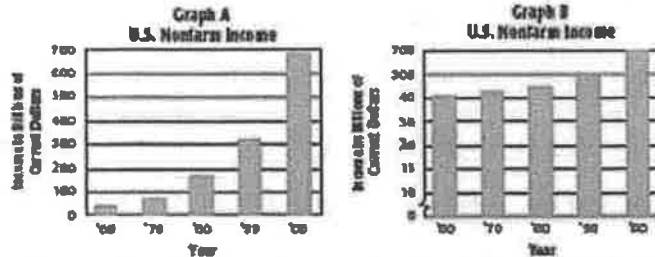
On a scale of 1 – 5 (1: Weak, 5: Strong) rate yourself on this section of math: 1 2 3 4 5

Unit: Knowledge of Statistics

Objective: Analyze data and recognize the misuses of data

Examples:

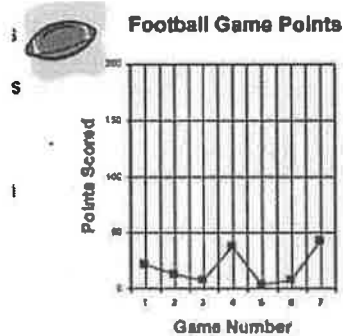
- Graphs can be misleading for many reasons: No title; the scale does not include 0; there are no labels on either axis; the intervals on a scale are not equal; or the size of the graphics misrepresents the data.



The bar graphs above show the total US National Income (nonfarm). Which graph is misleading? Explain.

- Graph B is misleading because the scale on the vertical axis does not have equal intervals. It makes the income appear to be slower.

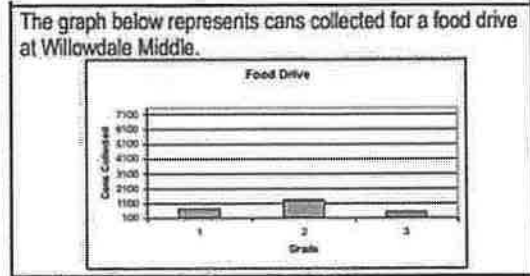
1. The graph represents points scored by the Baltimore Ravens during the 2003-2004 football season. What makes this graph misleading. Explain.



2. List 4 different situations that make a graph misleading.

3. Look at #1, what would you change to make this graph not misleading?

4. The graph represents cans collected for a food drive at a Middle School. How would you change the graph to better show the data and not be misleading?



Unit: Knowledge of Statistics

Objective: Determine the best choice of a data display for a given data set.

Examples:

- Different types of graphs are better suited for certain types of data.

Bar Graph – Use when comparing data (Ex. Football teams and # of wins)

Line Graph – Use when data is over time (Ex. Rainfall each month for 1 year)

Circle Graph (Pie Graph) – Use when data is dealing with \$ or % (Ex. Allowance – how you spend it)

Stem & Leaf Plot – Use to show individual data (Ex. Class test scores)

Back-to-Back Stem & Leaf Plot – Use when comparing 2 large sets of data & showing individual data scores

Directions: Look at the following situations and tell what type of graph would be the best choice to display the data. Choose BAR, LINE, CIRCLE, or STEM & LEAF.

1.) How the Federal Government spends each part of your tax dollar



2.) You are keeping track of your little sister's/brother's height from age 3 months to 5 years old

3.) Lengths of the 5 largest rivers in the world



4.) Number of points scored in each game during the 99-00 Season

Redskins: 35 50 27 38 24 20 21 26 21
48 17 28 23 20 17 28

Ravens: 10 20 17 19 11 8 10 41 3
34 23 41 31 31 22 3

5.)

Students who ride a bus	
YEAR	STUDENTS
2000	333
2001	297
2002	360
2003	365

6.)


# of Species at the Zoo	
ZOO	SPECIES
Los Angeles	350
Lincoln Park	290
Cincinnati	700
Bronx	530
Oklahoma City	600

On a scale of 1 – 5 (1: Weak, 5: Strong) rate yourself on this section of math: 1 2 3 4 5

Unit: Knowledge of Statistics

Objective: Compare the measures of central tendency (mean, median, mode) to determine which is most appropriate.

Examples:

	MEAN	MEDIAN	MODE
What is it?	Average	Middle #	# shown the MOST often
How to find it?	Sum of Data (+) # of Data Points (+)	Order data from least to greatest, then find the middle # 2 middle #'s - Average	Look at data & Find the # that appears the most.  2 modes - Bimodal
Most Useful when:	-- Data has no outliers Outliers are REALLY low & high #s	-- Data has outliers -- There are no large gaps in the middle of the data	-- Data has many identical (same) #s

Use the table at the right.

Find the mean, median, & mode of the data.

Mean: 488.3

Median: 150

Mode: None

Caribbean Islands			
Island	Area (Sq Mi)	Island	Area (Sq Mi)
Antigua	108	Martinique	425
Aruba	75	Puerto Rico	3,339
Barbados	166	Tobago	116
Curacao	171	Virgin Islands, UL	59
Dominica	290	Virgin Islands, US	134

Which measure of central tendency would be misleading in describing the size of the islands? Explain.

The mean could be misleading since the areas of all but one of the islands are less than that value.

Which measure would most accurately describe the data? Median

Book Sales: Use the table below that shows the number of books sold each day for 20 days to answer questions 4 – 5.



Book Sales Per Day			
23	18	23	15
24	16	0	11
19	10	13	17
12	23	11	16
36	24	12	27

1. Determine the mean, median and mode of the data.

2. Which measure of central tendency would be misleading in describing the book sales & which measure most accurately describes the data? Explain.

3. Michael & Melissa both claim to be earning a C average, 70% to 79%, in their Latin class. Use the table below to explain their reasoning and determine which student is earning a C average.

GRADES (%)							
	Test 1	Test 2	Test 3	Test 4	Test 5	Test 6	Test 7
Michael	80	76	73	70	40	25	10
Melissa	88	83	75	70	60	65	62

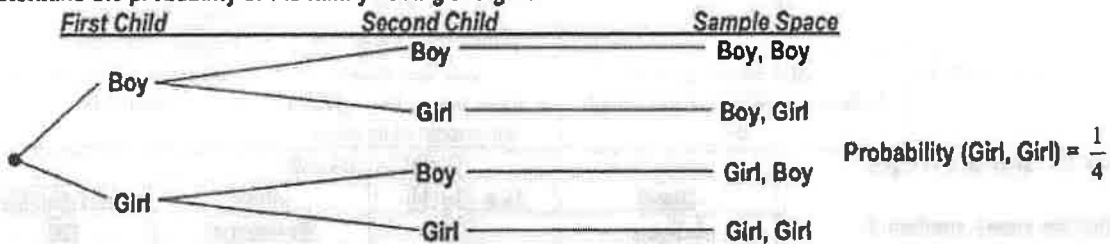
Unit: Knowledge of Probability

Objective: Identify a sample space and determine the number of outcomes using no more than 3 independent events.

Examples:

- **Sample Space** is a listing of all the possible outcomes in a probability experiment. One way to determine sample space is to draw a tree diagram.

A family has two children. Draw a tree diagram to show the sample space of the children's genders. Then determine the probability of the family having two girls.



- **FUNDAMENTAL COUNTING PRINCIPLE** is used to quickly determine the total number of possible outcomes. Multiply the number of possibilities for each event together.

An ice cream sundae at the Ice Cream Shoppe is made from one flavor of ice cream and one topping. For ice cream flavors, you can choose from chocolate, vanilla, and strawberry. For toppings, you can have hot fudge, butterscotch, caramel, and marshmallow. Determine the number of different sundaes that are possible.

# of ice cream flavors (Chocolate, Vanilla, Strawberry)	x	# of toppings (Hot Fudge, Butterscotch, Caramel, Marshmallow)
3	x	4
12 total possible outcomes		

<p>1. A certain type of kickboard scooter comes in silver, red, or purple with wheel sizes of 12mm or 180mm. Determine the total number of color-wheel size combinations.</p>	<p>2. Draw a tree diagram of the situation in #1 to show the sample space.</p>															
<p>3. The table shows the shirts, shorts and shoes in George's wardrobe. How many possible outfits can he choose consisting of one shirt, one pair of shorts and one pair of shoes?</p> <table border="1" data-bbox="446 1612 816 1734"> <thead> <tr> <th>SHIRTS</th> <th>SHORTS</th> <th>SHOES</th> </tr> </thead> <tbody> <tr> <td>Red</td> <td>Beige</td> <td>Black</td> </tr> <tr> <td>Blue</td> <td>Green</td> <td>Brown</td> </tr> <tr> <td>White</td> <td>Blue</td> <td></td> </tr> <tr> <td>Yellow</td> <td></td> <td></td> </tr> </tbody> </table>	SHIRTS	SHORTS	SHOES	Red	Beige	Black	Blue	Green	Brown	White	Blue		Yellow			<p>4. Determine the total number of outcomes by choosing a vowel from the word COMPUTER and a consonant from the word BOOK.</p>
SHIRTS	SHORTS	SHOES														
Red	Beige	Black														
Blue	Green	Brown														
White	Blue															
Yellow																

Unit: Knowledge of Probability

Objective: Make predictions and express probability of the results of a survey or simulation as a fraction, decimal, or percent. - A

Examples: Experimental probability can also be based on past performances and can be used to make predictions on future events.

In a survey, 100 people were asked to name their favorite Independence Day side dishes. What is the experimental probability of macaroni salad being someone's favorite dish?

There were 100 people surveyed and 12 chose macaroni salad, SO the experimental probability is $\frac{12}{100} = \frac{3}{25}$.

SIDE DISH	# of People
Potato Salad	55
Green Salad Or vegetables	25
Macaroni salad	12
Coleslaw	8

Suppose 250 people attend the city's Independence Day barbecue. How many can be expected to choose macaroni salad as their favorite side dish?

Write a proportion. $\frac{3}{25} = \frac{x}{250}$ (Use the experimental probability in the proportion.)

Solve by using cross products. $25x = 3(250)$

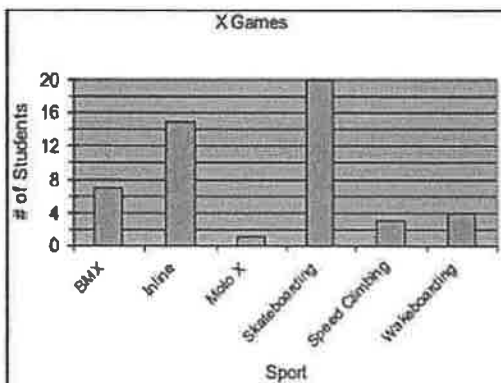
About 30 will choose macaroni salad. $x = 30$

1. Using the table in the example, what is the experimental probability of potato salad being someone's favorite dish?

2. Using the information in example and question 1, about how many people can be expected to choose potato salad as their favorite dish if 400 attend the barbeque?

3. The graph shows the results of a survey in which 50 students were asked to name their favorite X Game Sport.

- A. Suppose 500 people attend the X Games. How many can be expected to choose Inline as their favorite sport?
- B. Suppose 500 people attend the X Games. How many can be expected to choose speed climbing as their favorite sport?



Unit: Knowledge of Number Relationships & Computation

Objective: Determine equivalent forms of rational numbers expressed as fractions, decimals, percents, and ratios. - A

Examples:

To write a decimal as a fraction, divide the numerator of the fraction by the denominator.

Use a power of ten in the denominator to change a decimal to a fraction.

Write $\frac{5}{9}$ as a decimal.

$$\begin{array}{r} 0.555 \\ 9 \overline{) 5.000} \\ \underline{-45} \\ 50 \\ \underline{-45} \\ 50 \\ \underline{-45} \end{array} = 0.\overline{5} \text{ because 5 repeats forever.}$$

Write 0.32 as a fraction in simplest form.

$$0.32 = \frac{32}{100} = \frac{+4}{+4} = \frac{8}{25}$$

5/1.) Write 0.735353535... using bar notation to represent the repeating decimal.

2.) Write $\frac{3}{5}$ as a decimal.

3. Write 0.94 as a fraction in simplest form.

4. There were 6 girls and 18 boys in Mr. Johnson's math class. Write a ratio of the # of girls to the # of boys in fraction form. Then write the fraction as a repeating decimal.

Unit: Knowledge of Number Relationships & Computation

Objective: Determine equivalent forms of rational numbers expressed as fractions, decimals, percents, and ratios. - B

Examples:

A RATIO is a comparison of two numbers by division. When a ratio compares a number to 100, it can be written as a PERCENT. To write a ratio or fraction as a percent, find an equivalent fraction with a denominator of 100. You can also use the meaning of percent to change percents to fractions.

Write $\frac{19}{20}$ as a percent.

$$\frac{19}{20} \cdot \frac{5}{5} = \frac{95}{100} = 95\% \quad \text{Since } 100 \div 20 = 5, \text{ multiply the numerator and denominator by 5.}$$

Write 92% as a fraction in simplest form.

$$\frac{92}{100} = \frac{+4}{+4} = \frac{23}{25}$$

Write 92% as a decimal. Move decimal two places to the left. Add zeros if needed. 92.0% = 0.92

Write 0.4 as a percent. Move decimal two places to the right. Add zeros if needed. 0.4 = 40%

1.) Write $\frac{7}{25}$ as a percent and decimal.

2.) Write 19% as a decimal and fraction in simplest form.

3. Ms. Crest surveyed her class and found that 15 out of 30 students brushed their teeth more than twice a day. Write this ratio as a fraction in simplest form, then write it as a % and a decimal.

4. A local retail store was having a sale and offered all their merchandise at a 25% discount. Write this percent as a fraction in simplest form, then write it as a decimal.

Unit: Knowledge of Number Relationships & Computation

Objective: Compare, order, and describe rational numbers.

Examples:

- **RATIONAL** numbers include fractions, decimal, and percents. To **COMPARE** or **ORDER** rational numbers, they must be in the same form (all fraction or all decimals, or all %s)

Example: Order 0.6, 48%, and $\frac{1}{2}$ from least to greatest.

Step 1 – Change all to decimals. 0.6 48% = 0.48 $\frac{1}{2} = 0.5$

Step 2 – Compare decimals & Order. 0.48, 0.5, 0.6

Step 3 – Write using original form. 48%, $\frac{1}{2}$, 0.6

1.) Order from least to greatest.

22%, 0.3, $\frac{1}{5}$

2.) Order from least to greatest.

0.74, $\frac{3}{4}$, 70%

3.) Replace with <, >, or =.

$\frac{7}{12}$ 58%

4.) Which is the largest?

$1\frac{3}{8}$ $1\frac{3}{10}$ $1\frac{4}{9}$

5.) According to the Pet Food Manufacturer's Association, 11 out of 25 people own large dogs and 13 out of 50 medium dogs. Do more fraction of people own large or medium dogs?



6.) Your PE teacher asked you to run for specific time period. You ran 0.6 of the time. Two of your friends ran $\frac{7}{10}$ and 72% of the time. Order the amount of time you and your friends ran from least to greatest.

On a scale of 1 – 5 (1: Weak, 5: Strong) rate yourself on this section of math: 1 2 3 4 5

Unit: Knowledge of Number Relationships & Computation

Objective: Add, subtract, multiply and divide integers. - A

Examples:

ADDITION INTEGER RULES:

For integers with the same sign:

- The sum of two positive integers is **POSITIVE**.
- The sum of two negative integers is **NEGATIVE**.

For integers with different signs, subtract their absolute value. The sum is:

- **Positive** IF the positive integer has the greater absolute value.
- **Negative** IF the negative integers has the greater absolute value.

Examples:

$$-6 + (-3) = \text{add keep the sign} = -9$$

$$-34 + (-21) = \text{add keep the sign} = -55$$

$$8 + (-7) = \text{subtract keep the sign of the higher} = 1$$

$$-5 + 4 = \text{subtract keep the sign of the higher} = -1$$

SUBTRACTION INTEGER RULES:

- Keep the first number the same
- Switch the subtraction sign to **ADDITION**
- Change the second number to its opposite. Opposite: -6 to 6
- Follow Addition rules above.

Examples:

$$6 - 9 = 6 + (-9) = -3$$

$$-10 - (-12) = -10 + 12 = 2$$

$$-3 - 7 = -3 + (-7) = -10$$

$$1 - (-2) = 1 + 2 = 3$$

1.) Add: $2 + (-7)$

2.) Subtract: $-13 - 8$

3.) Evaluate $a - b$ if $a = -2$ and $b = -7$

4.) Evaluate $x + y + z$ if $x = 3$, $y = -5$, and $z = -2$

5.) In Mongolia the temperature can dip down to -45°C in January. The temperature in July may reach 40°C . What is the temperature range in Mongolia?

6.) Write an addition expression to describe skateboarding situation. Then determine the sum.

Hank starts at the bottom of a half pipe 6 feet below street level. He rises 14 feet at the top of his kickturn.

Unit: Knowledge of Number Relationships & Computation

Objective: Add, subtract, multiply and divide integers. - B

Examples:

MULTIPLYING & DIVIDING INTEGER RULES:

- Two integers with DIFFERENT signs the answer is NEGATIVE.
- Two integers with SAME signs the answer is POSITIVE.

Examples:

$5(-2) = 5$ times -2 , the signs are different so the answer will be negative = -10

$(-6)(-9) =$ the signs are the same so the answer will be positive = 54

$30 + (-5) =$ the signs are different so the answer will be negative = -6

$-100 + (-5) =$ the signs are the same so the answer will be positive = 20

1.) Multiply: $-14(-7)$	2.) Divide: $350 \div (-25)$
3.) Evaluate if $a = -3$ and $c = 5$ $-3ac$	4.) Evaluate if $d = -24$, $e = -4$, and $f = 8$ $\frac{de}{f}$
5.) A computer stock decreased 2 points each hour for 6 hours. Determine the total change in the stock value over the 6 hours.	6.) A submarine descends at a rate of 60 feet each minute. How long will it take it to descend to a depth of 660 feet below the surface?

On a scale of 1 – 5 (1: Weak, 5: Strong) rate yourself on this section of math: 1 2 3 4 5

Unit: Knowledge of Number Relationships & Computation

Objective: Add, subtract, and multiply positive fractions and mixed numbers. - A

Examples:

- To add unlike fractions (fractions with different denominators), rename the fractions so there is a common denominator.

$$\text{Add: } \frac{1}{6} + \frac{2}{5} =$$
$$\frac{5}{30} + \frac{12}{30} = \frac{17}{30}$$

$$\frac{1}{6} = \frac{1 \cdot 5}{6 \cdot 5} = \frac{5}{30}$$

$$\frac{2}{5} = \frac{2 \cdot 6}{5 \cdot 6} = \frac{12}{30}$$

$$\text{Add: } 12\frac{1}{2} + 8\frac{2}{3} =$$

$$12\frac{1}{2} = 12\frac{1 \cdot 3}{2 \cdot 3} = 12\frac{3}{6}$$

$$8\frac{2}{3} = 8\frac{2 \cdot 2}{3 \cdot 2} = 8\frac{4}{6}$$

$$12\frac{3}{6} + 8\frac{4}{6} = 20\frac{7}{6}$$

$\frac{7}{6}$ is improper so we must change it to proper. 7 divided by 6 = $1\frac{1}{6}$

$$20 + 1\frac{1}{6} = 21\frac{1}{6}$$

Add.

1. $\frac{1}{3} + \frac{1}{9}$

2. $2\frac{1}{2} + 2\frac{2}{3}$

3. The quiche recipe calls for $2\frac{3}{4}$ cups of grated cheese. A recipe for quesadillas requires $1\frac{1}{3}$ cups of grated cheese. What is the total amount of grated cheese needed for both recipes?

4. You want to make a scarf and matching hat. The pattern calls for $1\frac{7}{8}$ yards of fabric for the scarf and $2\frac{1}{2}$ yards of fabric for the hat. How much fabric do you need in all?

Unit: Knowledge of Number Relationships & Computation

Objective: Add, subtract, and multiply positive fractions and mixed numbers. - B

Examples:

- To subtract unlike fractions (fractions with different denominators), rename the fractions so there is a common denominator.

$$\text{Subtract: } \frac{7}{8} - \frac{1}{2} = \frac{7}{8} - \frac{4}{8} = \frac{3}{8} \quad \frac{7}{8} = \frac{7 \cdot 1}{8 \cdot 1} = \frac{7}{8} \quad \frac{1}{2} = \frac{1 \cdot 4}{2 \cdot 4} = \frac{4}{8} \quad \frac{7}{8} - \frac{4}{8} = \frac{3}{8}$$

$$\text{Subtract: } 5\frac{3}{4} - 2\frac{1}{3} = 5\frac{3}{4} = 5\frac{3 \cdot 3}{4 \cdot 3} = 5\frac{9}{12} \quad 2\frac{1}{3} = 2\frac{1 \cdot 4}{3 \cdot 4} = 2\frac{4}{12}$$

$$5\frac{9}{12} - 2\frac{4}{12} = 3\frac{5}{12}$$

****Note:** If you have to borrow from the whole number change to improper fractions, find a common denominator, subtract, and then change back to proper fractions.

Subtract.

1. $\frac{9}{10} - \frac{1}{10}$

2. $5\frac{3}{8} - 4\frac{11}{12}$

3. Melanie had $4\frac{2}{3}$ pounds of chopped walnuts. She used $1\frac{1}{4}$ pounds in a recipe. How many pounds of chopped walnuts did she have left?

4. Lois has $3\frac{1}{3}$ pounds of butter. She uses $\frac{3}{4}$ pound in a recipe. How much does she have left? *Hint: Change to improper fractions first.

Unit: Knowledge of Number Relationships & Computation

Objective: Add, subtract, and multiply positive fractions and mixed numbers. - C

Examples:

- To multiply fractions – Multiply the numerators & denominators.
- Be sure to change mixed numbers to improper fractions before multiplying.

$$\frac{1}{3} \cdot \frac{5}{8} = \frac{5}{24}$$

$$1\frac{1}{3} \cdot 3\frac{2}{5} = \frac{4}{3} \cdot \frac{17}{5} = \frac{68}{15} = 4\frac{8}{15}$$

****Remember:** Changing mixed numbers to improper fractions. $2\frac{3}{4} = 4 \cdot 2 + 3 = \frac{11}{4}$

$$1\frac{1}{3} \cdot 21 = \frac{4}{3} \cdot \frac{21}{1} = \frac{4 \cdot 21}{3 \cdot 1} = \frac{84}{3} = 28$$

1.) $\frac{2}{3} \cdot \frac{4}{5} =$

2.) $\frac{7}{3} \cdot 4\frac{1}{2} =$

3.) $2\frac{1}{2} \cdot 2\frac{1}{3} =$

4.) $3 \cdot 5\frac{2}{9} =$

5.) Anna wants to make 4 sets of curtains. Each set requires $5\frac{1}{8}$ yards of fabric. How much fabric does she need?

6.) One sixth of the students at a local college are seniors. The number of freshmen students is $2\frac{1}{2}$ times that amount. What fraction of the students are freshmen?

THE UNIVERSITY OF CHICAGO
 DEPARTMENT OF MATHEMATICS
 COURSE 581: ALGEBRA
 LECTURE 10: THE RING OF INTEGERS
 AND THE RING OF ALGEBRAIC INTEGERS

In this lecture, we will study the ring of integers of a number field. We will see that this ring is a Dedekind domain, and we will discuss the unique factorization of ideals into prime ideals. We will also study the ring of algebraic integers, which is the integral closure of the integers in a number field.

Let K be a number field of degree n over \mathbb{Q} . The ring of integers of K , denoted by \mathcal{O}_K , is the set of all elements $\alpha \in K$ such that α is integral over \mathbb{Z} . In other words, α is a root of a monic polynomial with coefficients in \mathbb{Z} .

The ring \mathcal{O}_K is a Dedekind domain, which means that it is a Noetherian integral domain in which every nonzero prime ideal is maximal. This implies that every nonzero ideal in \mathcal{O}_K can be uniquely factored into a product of prime ideals.

The ring of algebraic integers, denoted by $\overline{\mathbb{Z}}$, is the integral closure of \mathbb{Z} in \mathbb{C} . It consists of all algebraic integers in \mathbb{C} . The ring $\overline{\mathbb{Z}}$ is a Dedekind domain, and it is the integral closure of \mathbb{Z} in \mathbb{C} .

The ring of integers \mathcal{O}_K is a subring of $\overline{\mathbb{Z}}$. In fact, \mathcal{O}_K is the integral closure of \mathbb{Z} in K . This means that \mathcal{O}_K is the largest subring of K that is integral over \mathbb{Z} .

The ring \mathcal{O}_K is a free \mathbb{Z} -module of rank n . This means that there exists a \mathbb{Z} -basis for \mathcal{O}_K consisting of n elements. This basis is called an integral basis for K .

The discriminant of K , denoted by Δ_K , is the discriminant of the integral basis. It is a nonzero integer that depends only on K . The discriminant Δ_K is related to the volume of the fundamental parallelepiped of the lattice \mathcal{O}_K .

The ring \mathcal{O}_K is a local principal ideal domain (PID) at each prime \mathfrak{p} . This means that the localization $\mathcal{O}_{K, \mathfrak{p}}$ is a PID. This implies that every ideal in $\mathcal{O}_{K, \mathfrak{p}}$ is principal.

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